00 - Ismétlés

2016. szeptember 13.
20:09

vektor, skalár

$$\nabla T = \text{grad}\, T = \Big(\underbrace{\frac{\partial}{\partial x}}_{\underline{i}}, \underbrace{\frac{\partial}{\partial y}}_{j}, \underbrace{\frac{\partial}{\partial z}}_{\underline{k}}\Big) \cdot T(x,y,z) =$$

$$= \left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z}\right) \leftarrow \text{vektor}$$

$$\left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}\right)$$

$$\left(\frac{\partial T}{\partial x}, \partial\right)$$

$$\underline{q} = -\lambda \nabla T = (q_x, q_y, q_z) = \begin{pmatrix} -\lambda_x \frac{\partial T}{\partial x} \\ -\lambda_y \frac{\partial T}{\partial y} \\ -\lambda_z \frac{\partial T}{\partial z} \end{pmatrix} \text{tr}$$

hőáramsűrűség — vektortér

adott f vektortér potenciálja φ, ha $f = \nabla \varphi$

01 - divergencia

2016. szeptember 16.
8:11

$$\text{skalártér} \xrightarrow{\text{grad}} \text{vektortér} \xrightarrow{\text{div}} \text{skalártér}$$

$T \qquad\qquad q$

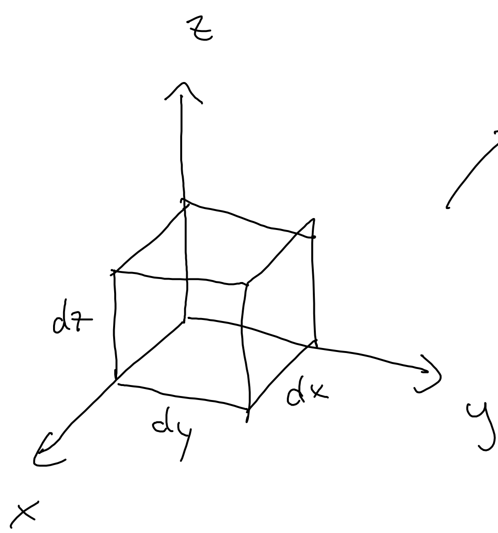
Divergencia = a vektortér kifelé mutató többlet áramának egy térfogati egységre vetített értéke

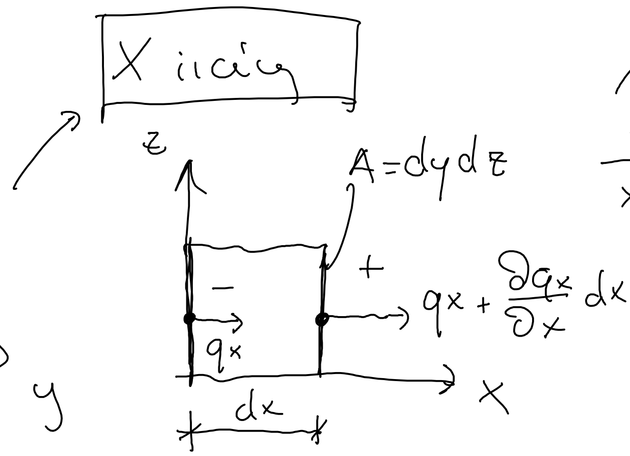
$$q = \begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix}$$

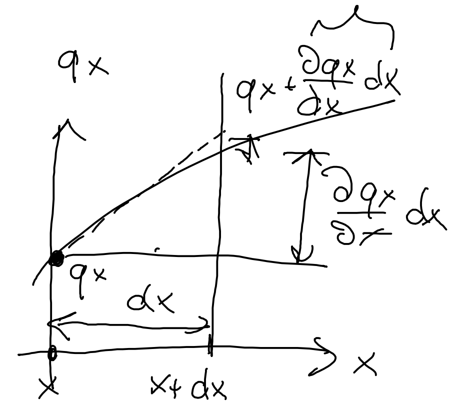
$$\operatorname{div} q = \nabla q = \underbrace{\left(\frac{\partial}{\partial x}\ \frac{\partial}{\partial y}\ \frac{\partial}{\partial z}\right)}_{\text{vektor}} \cdot \overbrace{\begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix}}^{\text{vektor}} = \underbrace{\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}}_{\text{skalár}}$$

$$\boxed{\begin{aligned} a \cdot b &= |a||b| \cdot \cos\alpha \\ a \cdot b &= a_1 b_1 + a_2 b_2 + a_3 b_3 \end{aligned}}$$

skaláris szorzat







$dx \to 0$

$$\left(\cancel{q_x} + \frac{\partial q_x}{\partial x} dx - \cancel{q_x}\right) dy\,dz + \left(\cancel{q_y} + \frac{\partial q_y}{\partial y} dy - \cancel{q_y}\right) dx\,dz +$$

$$+ \left(\cancel{q_z} + \frac{\partial q_z}{\partial z} dz - \cancel{q_z}\right) dx\,dy = \left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right) dx\,dy\,dz$$

$$+\left(\cancel{q_z}+\frac{\partial q_z}{\partial z}dz-\cancel{q_z}\right)dx\,dy=\left(\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}+\frac{\partial q_z}{\partial z}\right)\underbrace{dx\,dy\,dz}_{=dV}$$

$$=\operatorname{div.} q\, dV=\nabla q\, dV$$

ált. transzportegyenlet

$\boxed{\text{ext. mennyiség időegység alatti megv. az elemi térfogatban}} = \boxed{\text{vezetéses ki-és be-áramlás előjeles összege az elemi térfogatban}} + \cancel{\boxed{\text{forrás/nyelő elemi térf.}}} + \cancel{\boxed{\text{konvektív áram. ki-be előjeles össz. el. térf.}}}$

$\cancel{= 0}$ $\qquad$ $= 0$

$$-\operatorname{div} q = 0$$

$$-\operatorname{div}(\operatorname{grad} T) = 0$$

$$-\operatorname{div}\left(-\lambda_x \frac{\partial T}{\partial x}, -\lambda_y \frac{\partial T}{\partial y}, -\lambda_z \frac{\partial T}{\partial z}\right) = 0$$

$$\nabla\left(\lambda_x \frac{\partial T}{\partial x}, \lambda_y \frac{\partial T}{\partial y}, \lambda_z \frac{\partial T}{\partial z}\right) = 0$$

$$\begin{vmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{vmatrix} \left(\lambda_x \frac{\partial T}{\partial x}, \lambda_y \frac{\partial T}{\partial y}, \lambda_z \frac{\partial T}{\partial z}\right) = 0$$

ha $\lambda_x = \lambda_y, \lambda_z$

$$\to \boxed{\lambda \frac{\partial^2 T}{\partial x^2} + \lambda \frac{\partial^2 T}{\partial y^2} + \lambda \frac{\partial^2 T}{\partial z^2} = 0}$$

stac. hővezetés differenciál-egyenlete

$T(x, y, z)$

$$\sim \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2} = 0$$

Laplace egyenlet $= \Delta T$

$$\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=0\right)$$

elnevezet $= \Delta$

$$\left(\frac{\partial^2}{\partial x^2},\ \frac{\partial^2}{\partial y^2},\ \frac{\partial^2}{\partial z^2}\right)$$

$$\lambda_x \frac{\partial^2 T}{\partial x^2}=0$$

$$\left[\frac{W}{m\cancel{K}}\right]\frac{[\cancel{K}]}{[m^2]}=\left[\frac{W}{m^3}\right]$$

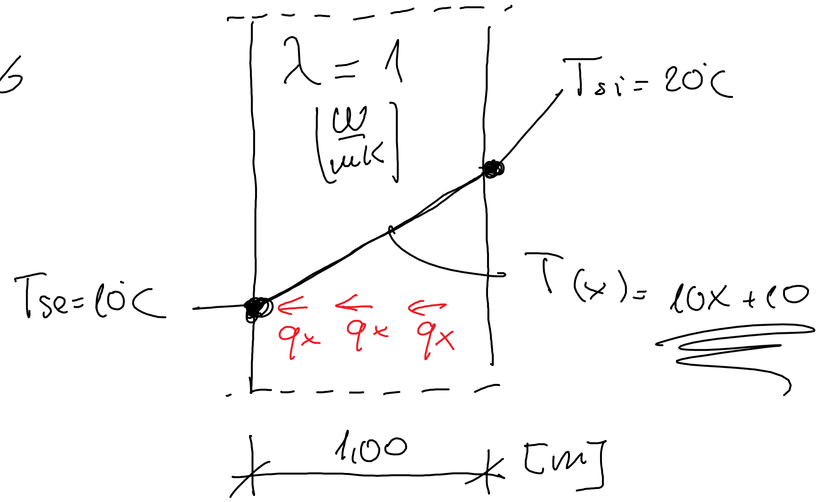
W ← hő, ext. mennyiség

m^3 ← elemi térfogatra

03 - Hőfokesési görbe, U érték számítás

2016. szeptember 16.
8:22

$$\lambda \frac{\partial^2 T}{\partial x^2} + \lambda \frac{\partial^2 T}{\partial y^2} + \lambda \frac{\partial^2 T}{\partial z^2} = \emptyset$$



(1) 1D

$$\lambda \frac{\partial^2 T}{\partial x^2} = \emptyset$$

$T(x) = ? = \underline{a \cdot x + b}$ ált. megoldás

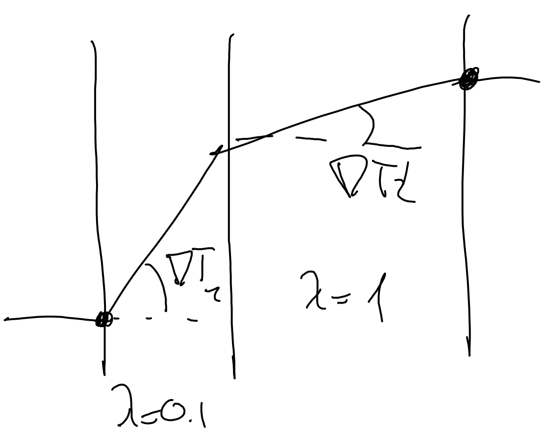
$\frac{\partial T}{\partial x} = ? = \frac{10K}{1m}$

$q_x = ? = -\lambda \nabla T = -1 \cdot \frac{10\,K}{m} \frac{W}{mK} = -10 \frac{W}{m^2K}$

(2) $T(x=0) = 10°C \rightarrow T(x=0) = a \cdot 0 + b = 10°C \rightarrow b = 10°C$

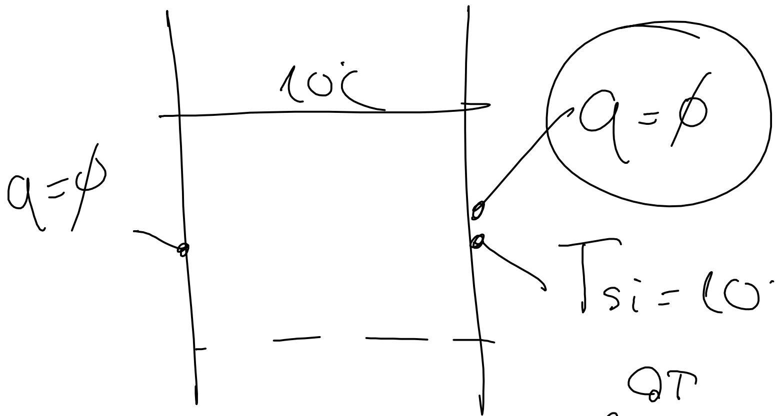
$T(x=1) = 20°C \rightarrow T(x=1) = a + 10 = 20°C \rightarrow a = 10$

$q(x) = -10 \rightarrow \text{div. } q = ? = \emptyset$

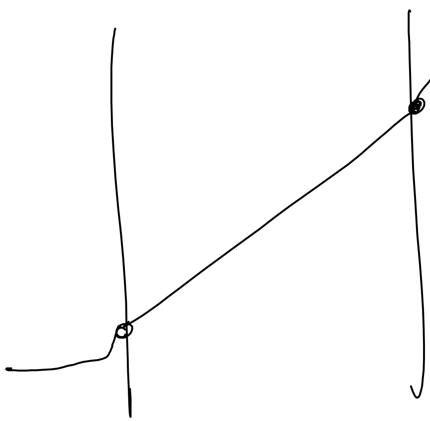


$$\lambda_1 \cdot \nabla T_1 = \lambda_2 \nabla T_2$$

$$q_1 = q_2$$



$-\lambda \frac{\partial T}{\partial n}$



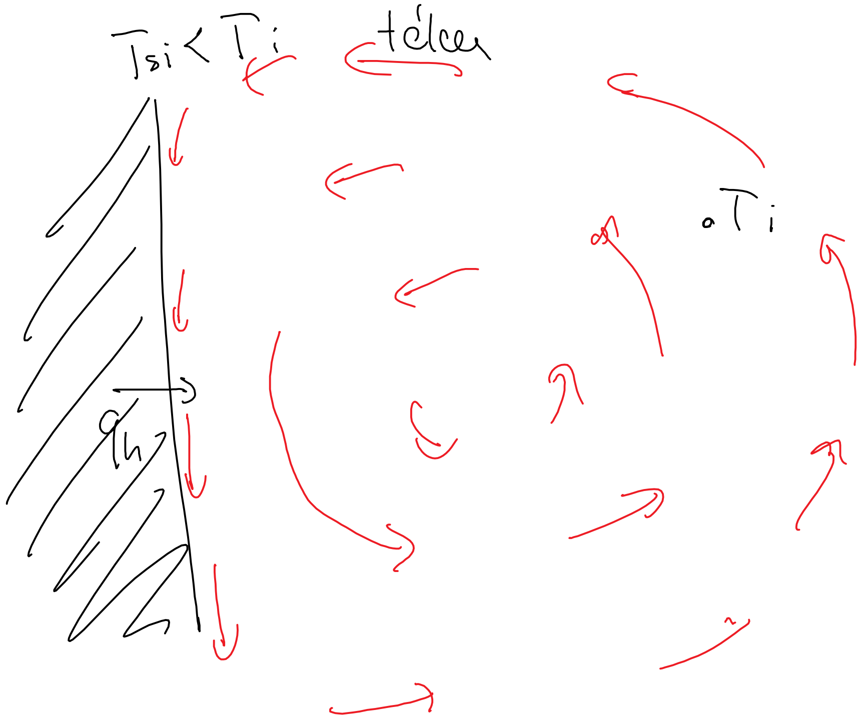
$$q_{si} = h_i \cdot (T_\infty - T_{si})$$

$8 \left[\frac{W}{m^2K}\right]$

- $T_\infty = 20°C$

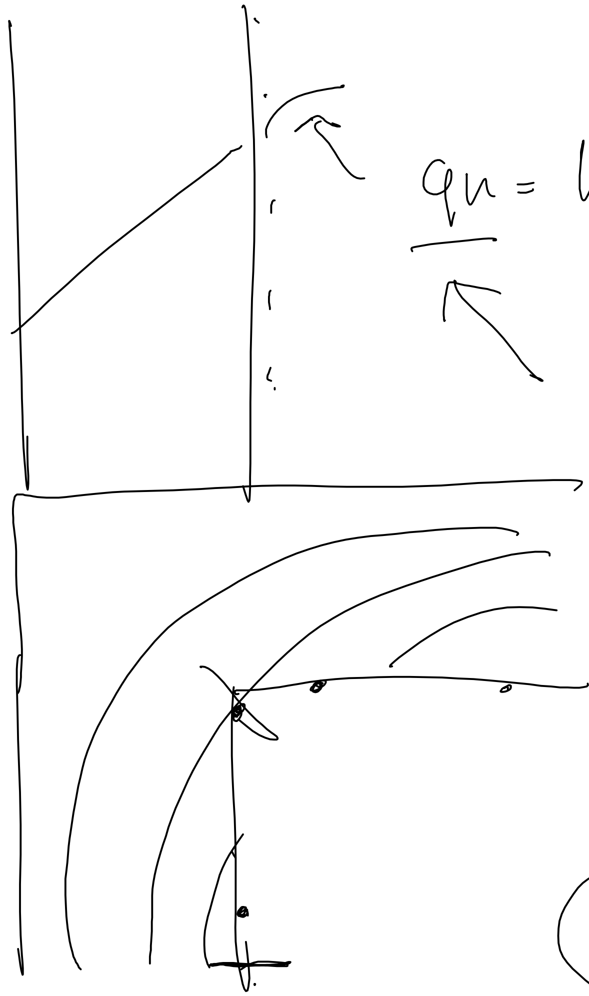
$\parallel$

$T_{air,in} = 20°C$



$$q_h \simeq h_i \underbrace{(T_\infty - T_{si})}_{\Delta T_s}$$

$$q_n = h_i (T_\infty - T_{si})$$



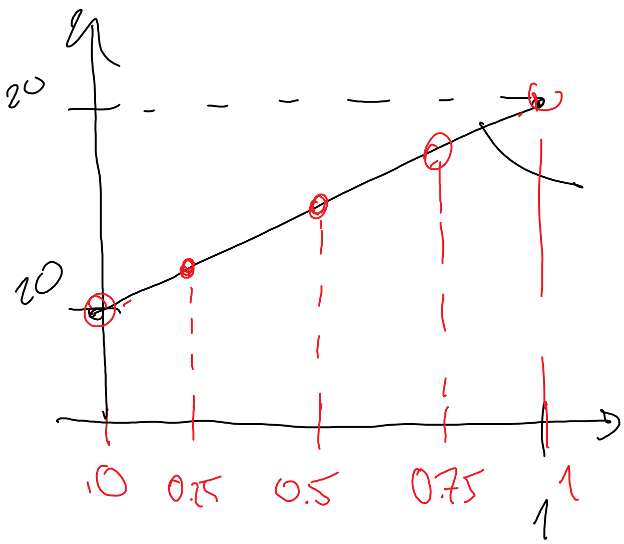
T_∞

04 - Analtikus vs. numerikus megoldás

2016. szeptember 16.
8:28

$T(x) = ax + b = 10x + 10$ ← analitikus

v. zárt képlet
szerinti

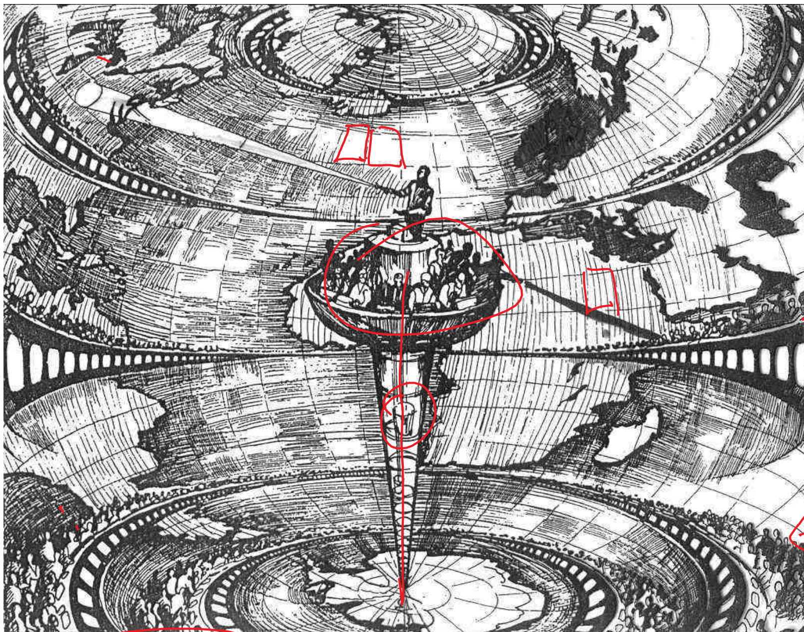


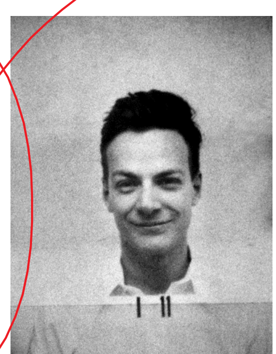
$T(x) = 10x + 10$

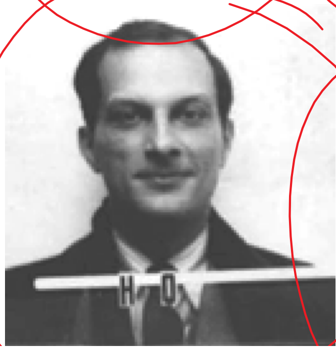
x	0	0.25		1
T	10	12.5	...	20
	10	12.49598		20.0562..

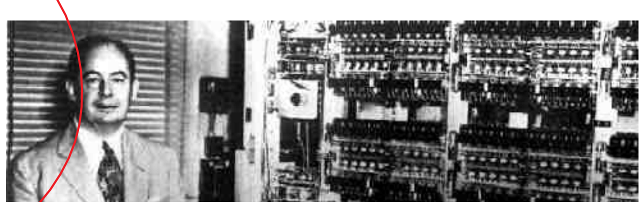
05 - numerikus módszerek

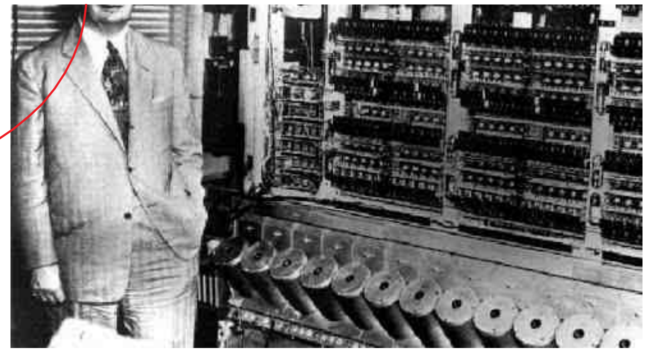
2016. szeptember 16.
8:28

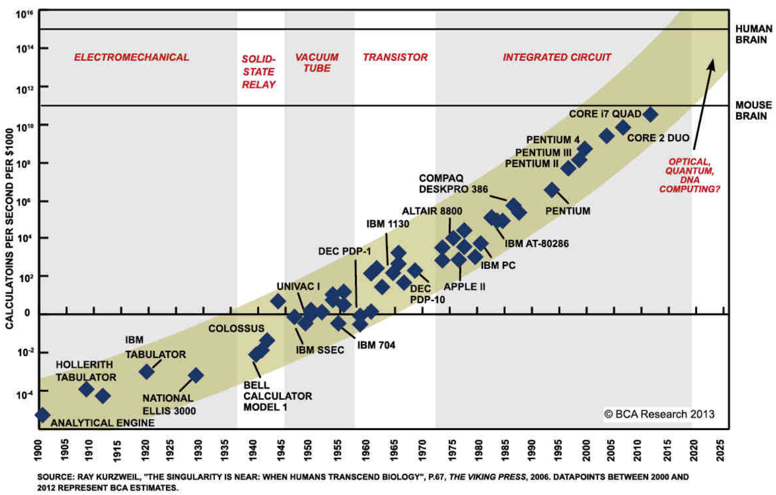












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Moor törvénye